Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

Note on Problem 33. By Henry Heaton, Atlantic, Iowa.

The result obtained in the published solution of this problem cannot be correct.

The area of the regular pentagon is $3.6327a^2$. The area of each of the infinite number of regular polygons whose apothem is a and number of sides greater than five, is less than $3.6327a^2$, while that of only two, the square and triangle, is greater. Hence the average area of all regular polygons with apothem a is less than $3.6327a^2$. Hence the result obtained in the published solution ($3.8693a^2$) is too large.

In a similar manner it may be shown that any result larger than $a^2\pi$ is too large, while it is evident that any result smaller than that is too small.

37. Proposed by HENRY HEATON, M. Sc., Atlantic, Iowa.

Required the average area of all triangles two of whose sides are a and b.

I. Solution by the PROPOSER.

It is well known that every triangle consists of six parts, three sides and three angles, and one side with any two other parts determines the triangle.

In constructing this triangle we may use all possible values, first, of the included angle C, second, of the third side, c, third, of the angle A, and fourth, of the angle B. This gives us four cases.

I. Put angle $C=\theta$. Then $A_1=\frac{ab}{2}\int_0^\pi \sin\theta d\theta \div \int_0^\pi d\theta=\frac{ab}{\pi}$.

II. Put side $c=x$. Then $A_2=\frac{1}{4}\int_{a-b}^{a+b}[(a+b)^2-x^2]^{\frac{1}{2}}[x^2-(a-b)^2]^{\frac{1}{2}}dx$.

$$\div\int_{a-b}^{a+b}dx=\frac{a+b}{12b}\left\{(a^2+b^2)E\left[\left(\frac{2\sqrt{ab}}{a-b}\right),\tfrac{1}{2}\pi\right]-(a-b)^2F\left[\left(\frac{2\sqrt{ab}}{a+b}\right),\tfrac{1}{2}\pi\right]\right\}.$$

(To integrate this expression put $x=[(a+b)^2-4ab\sin\theta]^{\frac{1}{2}}$).

III. Put angle $A=\theta$, b being $<a$, then

$$A_3=\tfrac{1}{2}b\int_0^\pi[b\cos\theta+(a^2-b^2\sin^2\theta)^{\frac{1}{2}}]\sin\theta d\theta\div\int_0^\pi d\theta=\frac{ab}{2\pi}+\left(\frac{a^2--b^2}{4\pi}\right)\log_e\left(\frac{a+b}{a-b}\right).$$

IV. Put angle $B=\theta$. For every value of B there are two triangles whose average arc is $\frac{1}{2}a^2\sin\theta\cos\theta$. Hence,

$$A_4=\tfrac{1}{2}a^2\int_0^{\sin^{-1}\frac{b}{a}}\sin\theta\cos\theta d\theta\div\int_0^{\sin^{-1}\frac{b}{a}}d\theta=\tfrac{1}{4}b^2\div\sin^{-1}\frac{b}{a}.$$

Corollary. If $b=a$, $A_1=a^2/\pi$, and $A^2=a^2/3$. These are double the values found in the solutions of problem 26, as they evidently should be. The values of A_3 and A_4 do not hold when $b=a$, for the reason that while the sum of the areas remains the same the number of triangles is reduced one-half at the moment that $b=a$.

II. **Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.**

Let x=third side, A=area, Δ=average area.

$$\therefore\ A=\tfrac{1}{4}\sqrt{(a+b)^2-x^2}\sqrt{x^2-(a-b)^2}.$$

$$\Delta=\int_{a-b}^{a+b}A\,dx\div\int_{a-b}^{a+b}dx=\frac{1}{2b}\int_{a-b}^{a+b}A\,dx.$$

Let $(a+b)^2-x^2=4aby^2$, $\dfrac{4ab}{(a+b)^2}=e^2$.

$$\therefore\ \Delta=\frac{2a^2b}{a+b}\int_0^1\frac{y^2\sqrt{1-y^2}\,dy}{\sqrt{1-e^2y^2}}=\tfrac{1}{2}a(a+b)\int_0^1\frac{y^2\sqrt{1-e^2y^2}\,dy}{\sqrt{1-y^2}}$$

$$-\frac{a(a-b)^2}{2(a+b)}\int_0^1\frac{y^2\,dy}{\sqrt{1-y^2}\sqrt{1-e^2y^2}}=\frac{a+b}{12b}\{(a^2+b^2)E(e)-(a-b)^2F(e)\}.$$

III. **Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics, Columbian University, Washington, D. C., and A. P. REED, Clarence, Missouri.**

The area of triangle is $\triangle=\frac{1}{2}ab\sin\theta$.

$$\text{Hence, average area}=\tfrac{1}{2}ab\int_0^{\pi}\sin\theta d\theta\div\int_0^{\pi}d\theta=\tfrac{1}{2}ab\Big[-\cos\theta\Big]_0^{\pi}\div\pi=\frac{ab}{\pi}.$$

38. **Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics, Drury College, Springfield, Missouri.**

Two arrows are sticking in a circular target : show that the chance that their distance is greater than the radius of the target is $3\sqrt{3}/4\pi$. [From *Todhunter's Integral Calculus*, page 335.]